GLOBAL ATTRACTORS OF NONAUTONOMOUS DISPERSE DYNAMICAL SYSTEMS AND DIFFERENTIAL INCLUSIONS.

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Abstract. Article is devoted to nonautonomous dynamical systems without uniqueness, which admit compact global attractor. In article is given the application of obtained results to differential equations without uniqueness, differential and difference inclusions.

Beginning with the classical works of N.Levinson (see for ex.[1]) a great number of works are devoted to the study of dissipative systems. The results for autonomous and periodical finite-dimensional dissipative systems are reflected in monograph [2-4].

In the works of J.Hale and his students many important results obtained earlier for ordinary differential equations are generalized on the functionally-differential equations. These results are reflected on monographs [5-6] and surveys [7-8].

In the last years a great number of works are devoted to the study of dissipative partial differential equations. Results, obtained in this field are reflected in monograph [9] and articles [10-11].

In the works mentioned above with the exception of only few were considered autonomous and periodical systems. In the works of D.N.Cheban (see for ex. [12-20]) are studied abstract dynamical systems (autonomous and nonautonomous) possessing the property of dissipativity. And it turned out that many principle facts established earlier for the periodical dissipative systems are the partial manifestation of general principles established for the arbitrary nonautonomous systems.

In the last years in the works of D.N.Cheban and D.S.Fakeeh were studied autonomous dissipative dynamical systems without uniqueness (see for ex. [21-22]).

The present article is devoted to the study of nonautonomous dissipative dynamical systems without uniqueness and their applications to the differential and difference inclusions.

§ 1. Global attractors of autonomous disperse dynamical systems.

Let (X, ρ) complete metric space, $\mathbb{S}$ is a group of real ($\mathbb{R}$) or integer ($\mathbb{Z}$) numbers, $\mathbb{S}_+ \subseteq \mathbb{T}$) is semigroup of additive group $\mathbb{S}$. If $A \subseteq X$ and $x \in X$ then will note $\rho(x, A)$ a distance from point x to set A i.e. $\rho(x, A) = \inf\{\rho(x, a) : a \in A\}$. Will note by $B(A, \varepsilon)$ the $\varepsilon-$ neighborhood of set A i.e. $B(A, \varepsilon) = \{x \in X : \rho(x, A) < \varepsilon\}$. Will note by $C(X)$ the family of all non-empty compact subsets of X. For every point $x \in X$ and number $t \in \mathbb{T}$ will put in correspondence closed compact subset $\pi(x,t) \in C(X)$ and so if $\pi(A,P) = \bigcup\{\pi(x,t) : x \in A, t \in P\}(P \subseteq \mathbb{T})$ then

1.1. $\pi(x,0) = x$ for all $x \in X$;

1.2. $\pi(\pi(x,t_1),t_2) = \pi(x,t_1+t_2)$ for all $x \in X$ and $t_1,t_2 \in \mathbb{T}$, if $t_2 \cdot t_2 > 0$;

1.3. $\lim\limits_{x\to x_0, t\to t_0} \beta(\pi(x,t),\pi(x_0,t_0)) = 0$ for all $x_0 \in X$ and $t_0 \in \mathbb{T}$, where $\beta(A,B) = \sup\{\rho(a,B) : a \in A\}$ semi-deviation of set $A \subseteq X$ from set $B \subseteq X$. In this case it is said [23] that is defined semi-dynamical system. Let $\mathbb{T} = \mathbb{S}$ and is fulfilled condition

1.4. If $p \in \pi(x,t)$ then $x \in \pi(p,-t)$ for all $x,p \in X$ and $t \in \mathbb{T}$ then it is said that is defined dynamical system (X,T,π) or dynamical systems without uniqueness.

Observation. Later on under the disperse dynamical system (X,T,π) we will understand a semidynamical system unless otherwise stated, i.e. we will consider, that $\mathbb{T} = \mathbb{S}_+$.

Let $\mathcal{M}$ is some family of subsets of X. We will call a dynamical systems $(X,\mathbb{T},\pi)$ $\mathcal{M}$-dissipative it there exists a bounded set $K \subseteq X$, such that for any $\varepsilon > 0$ and $M \in \mathcal{M}$ exists $L = L(\varepsilon, M) > 0$ such that $\pi^t M \subseteq B(K,\varepsilon)$ for every $t \geq L(\varepsilon,M)$, where $\pi^t M = \{\pi(x,t) = x \cdot t : x \in M\}$. In addition we will call set K as attractor for family $\mathcal{M}$. The most interesting for applications are cases when $\mathcal{M} = \{\{x\} : x \in X\}, \mathcal{M} = C(X), \mathcal{M} = \{B(x,\delta_x) : x \in X, \delta_x > 0 \text{ is fixed }\}$ or $\mathcal{M} = \mathbb{B}(X)$ (where $\mathbb{B}(X)$ family of all bounded subsets of X).

System (X,T,π) is called [21-22]:

-pointwise dissipative if there exists $K \in \mathbb{B}(X)$ such that for all $x \in X$

$$\lim_{t\to+\infty} \beta(x \cdot t, K) = 0; \tag{1.1}$$

-compactly dissipative if equality (1.1) holds uniformly by x on compacts from X;

-locally dissipative if for any point $p \in X$ exists $\delta_p > 0$ such that equality (1.1) holds uniformly by $x \in B(p,\delta_p)$;

-boundedly dissipative if equality (1.1) holds uniformly by x on every bounded subset from X.

In the study of dissipative systems are distinguished two cases when K is compact and bounded (but not compact). According to this will called system $(X,\mathbb{T},\pi)$ pointwise $k(b)-$ dissipative if $(X,\mathbb{T},\pi)$ is pointwise dissipative and set K appearing in (1.1) is compact (bounded). Analogously are defined the notions of compact $k(b)$ dissipativity and other types of dissipativity.

Let $(X,\mathbb{T},\pi)$ is compactly k dissipative and K is compact set being attractor for all compact subsets of X. Will set

$$J = \Omega(K) = \bigcap_{t\geq 0} \overline{\bigcup_{\tau\geq t} \pi^\tau K}. \tag{1.2}$$

It can be shown [21] that set J defined by equality (1.2), don't depends on the choice of attractor K, but is characterized only by properties of dynamical system $(X,\mathbb{T},\pi)$ itself. Set J is called center of Levinson of compact dissipative system $(X,\mathbb{T},\pi)$.

Will state some known facts, which will be necessary for us below.

Theorem 1.1. [21 − 22] *If $(X, \mathbb{T}, \pi)$ is compactly dissipative dynamical system and J is its center of Levinson then : 1. J is invariant, i.e. $\pi^t J = J$ for all $t \in \mathbb{T}$;*

2. J is orbitally stable , i.e. for any $\varepsilon > 0$ exists $\delta(\varepsilon) > 0$ such that $\rho(x, J) < \delta$ implies $\beta(xt, J) < \varepsilon$ for all $t \geq 0$;

3. J is attractor of family of all compact subsets of X;

4. J is maximal compact invariant set $(X, \mathbb{T}, \pi)$.

Continuous single-valued mapping $\varphi_x : \mathbb{T} \to X$ is called motion of disperse dynamical system $(X, \mathbb{T}, \pi)$ starting from point $x \in X$ if $\varphi_x(0) = x$ and $\varphi_x(t_2) \in \pi(\varphi_x(t_1), t_2 - t_1)$ for any $t_1, t_2 \in \mathbb{T}(t_2 > t_1)$.

Set of all motions $(X, \mathbb{T}, \pi)$ starting from point x is noted by Φ_x and $\Phi(\pi) = \bigcup\{\Phi_x : x \in X\}$.

Dynamical system $(X, \mathbb{T}, \pi)$ is called :

-locally completely continuous if for any point $p \in X$ exists $\delta_p > 0$ and $l_p > 0$ such that $\pi^{l_p} B(p, \delta_p)$ is relatively compact ;

-weakly dissipative, if there exists non-empty compact $K \subseteq X$ such that for any $x \in X$ and $\varphi_x \in \Phi_x$ be found $\tau = \tau(x, \varphi_x) > 0$ for which $\varphi_x(\tau) \in K$;

-trajectoricaly dissipative if there exists non-empty compact such that

$$\lim_{t \to +\infty} \rho(\varphi_x(t), K) = 0 \tag{1.3}$$

for all $x \in X$ and $\varphi_x \in \Phi_x$.

Theorem 1.2. [21 − 22] *Let $(X, \mathbb{T}, \pi)$ is trajectoricaly dissipative and $(X, \mathbb{T}, \pi)$ is locally completely continuous, then $(X, \mathbb{T}, \pi)$ is locally dissipative.*

Theorem 1.3. *Let $(X, \mathbb{T}, \pi)$ is weakly dissipative and locally completely continuous, then $(X, \mathbb{T}, \pi)$ is trajectoricaly dissipative.*

Proof. Let $(X, \mathbb{T}, \pi)$ weakly dissipative and K is such non-empty compact that for any $x \in X$ and $\varphi_x \in \Phi_x$ will be found $\tau = \tau(x, \varphi_x) > 0$ for which $\varphi_x(\tau) \in K$. By virtue of local completely continuity of $(X, \mathbb{T}, \pi)$ and compactness of K will be found $\delta_0 > 0$ and $l_0 > 0$ such that $\pi^{l_0} B[K, \delta_0]$ is relatively compact. Will set $M = \overline{\pi^{l_0} B[K, \delta_0]}$,

$$\eta(x, \varphi_x) = \sup\{t : \forall \tau \in [0, t], \varphi_x(\tau) \in B[K, \delta_0]\}$$

and

$$\gamma(x, \varphi_x) = \sup\{t : \forall \tau \in (\tau, t], \varphi_x(\tau) \notin B[K, \delta_0]\}$$

for $x \in M$. Will show that there exists $l > 0$ for which

$$\gamma(x, \varphi_x) - \eta(x, \varphi) \leq l \tag{1.4}$$

for all $x \in B[K, \delta_0]$. If we will suppose that it is wrong then will be found $x_n \in B[K, \delta_0]$ and $\varphi_{x_n} \in \Phi_{x_n}$ such that $\gamma(x_n, \varphi_{x_n}) - \eta(x_n, \varphi_{x_n}) = l_n \to +\infty$. Will set $\overline{x}_n = \varphi_{x_n}(\eta(x_n, \varphi_{x_n}) + l)$, then $\{\overline{x}_n\} \subseteq M$ and therefore sequence $\{\overline{x}_n\}$ may be considered convergent. Let $\overline{x} = \lim\limits_{n \to +\infty} \overline{x}_n$. From definition of $\overline{x}_n$ follows that

$$\rho(\varphi_{\overline{x}_n}(t), B[K, \delta_0]) > 0 \tag{1.5}$$

for all $t \in (0, l_n - l)$, where $\varphi_{\overline{x}_n}(t) = \varphi_{x_n}(t + \tau(x_n, \varphi_{x_n}) + l)$. Since $\overline{x}_n \to \overline{x}$, then according to theorem 0.1.1 [22] from sequence $\{\varphi_{\overline{x}_n}\}$ may be extracted subsequence converging uniformly on every compact from $\mathbb{T}$ to some motion $\varphi_{\overline{x}}$ of dynamical system $(X, \mathbb{T}, \pi)$. From (1.5) follows that

$$\rho(\varphi_{\overline{x}}(t), B[K, \delta_0]) \geq 0 \tag{1.6}$$

for all $t > 0$. Last relation contradicts with weak dissipativity of $(X, \mathbb{T}, \pi)$, which proves the required affirmation. Will note, that set $K_1 = \pi(M, [0, l])$ is non-empty and compact. Let $x \in X, \varphi_x \in \Phi_x$ and $\tau = \tau(x, \varphi_x) > 0$ is such that $\varphi_x(\tau) \in M$, then $\varphi_x(t) \in K_1$ for all $t \geq \tau + l_0$. Theorem is proved.

Consequence 1.4. *Let $(X, \mathbb{T}, \pi)$ is weakly dissipative and locally completely continuous, then $(X, \mathbb{T}, \pi)$ is locally dissipative.*

Lemma 1.5. [18, 21] *Let $M \in \mathcal{B}(X)$, then following conditions are equivalent :*

1. whatever are $\{x_k\} \subseteq M$ and $t_k \to +\infty$ the sequence $\{y_k\}(y_k \in \pi(x_k, t_k))$ is relatively compact ;

2. $\Omega(M) \neq \emptyset$, is compact, invariant and

$$\lim_{t \to +\infty} \beta(\pi^t(M), \Omega(M)) = 0; \tag{1.7}$$

3. exists non-empty compact $K \subseteq X$ such that

$$\lim_{t \to +\infty} \beta(\pi^t(M, K) = 0. \tag{1.8}$$

Will say that system $(X, \mathbb{T}, \pi)$ satisfies condition of Ladyzhenskaia, if whatever is set $M \in \mathcal{B}(X)$ one of the conditions 1-3 of lemma 1.5 is fulfilled .

Theorem 1.6. $[21 - 22]$ *Let $(X, \mathbb{T}, \pi)$ satisfies condition of Ladyzhenskaja, then following conditions are equivalent:*

1. exists bounded set $B_1 \subseteq X$ such that for any $x \in X$ will be found $\tau \geq 0$ such that $\pi(x, t) \subseteq B_1$ for all $t \geq \tau$;

2. exists bounded set $B_2 \subseteq X$ such that for any $x \in X$ will be found $\tau \geq 0$ such that $\pi(x, \tau) \subseteq B_2$;

3. exists non-empty compact $K_1 \subseteq X$ such that $\lim\limits_{t \to +\infty} \beta(\pi(x, t), K_1) = 0$ for all $x \in X$;

4. exists non-empty compact $K_2 \subseteq X$ such that for all $\varepsilon > 0$ and $x \in X$ exists $\tau(\varepsilon, x) > 0$ such that $\pi(x, \tau) \subseteq B(K, \varepsilon)$;

5. exists non-empty compact set $K_3 \subseteq X$ such that for any bounded set $B \in B(X)$ holds equality

$$\lim_{t \to +\infty} \beta(\pi^t B, K_3) = 0; \tag{1.9}$$

6. exists bounded set B_3 such that for any bounded set $B \in B(X)$ exists $L = L(B) > 0$, such $\pi^t B \subseteq B_3$ for all $t \geq L(B)$.

Lemma 1.7. [21] *Dynamical system $(X, \mathbb{T}, \pi)$ is compactly dissipative if and only if the following conditions holds :*

a. $(X, \mathbb{T}, \pi)$ is trajectoricaly dissipative ;

b. $\Sigma^+(M) = \bigcup\{\pi^t M : t \geq 0\}$ is reactively compact for all $M \in C(X)$.

Theorem 1.8. *Let $(X, \mathbb{T}, \pi)$ satisfies condition of Ladyzhenskaja. For the fulfilling of one of conditions 1-6 of theorem 1.6 is necessary and sufficient the existence of bounded set $B_0 \subset X$ such that for any $x \in X$ and $\varphi_x \in \Phi_x$ will be found $\tau = \tau(x, \varphi_x) \geq 0$ for which $\varphi_x(\tau) \in B_0$.*

Proof. Necessity of theorem is evident. Will show its sufficiency. Let B_0 is bounded set appearing in theorem 1.7, $x \in X, \varphi_x \in \Phi x$ and $\tau = \tau(x, \varphi_x) \geq 0$ such that $\varphi_x(\tau) \in B_0$. Since $(X, \mathbb{T}, \pi)$ satisfies condition of Ladyzhenskaja then for bounded set B_0 will be found compact K such that

$$\lim_{t \to +\infty} \beta(\pi^t B_0, K) = 0. \tag{1.10}$$

From (1.10) and inclusions $\varphi_x(\tau) \in B_0$ and $\varphi_x(t + \tau) \in \pi^t B_0$ for all $t \geq 0$ following equality

$$\lim_{t \to +\infty} \rho(\varphi_x(t), K) = 0, \tag{1.11}$$

i.e. $(X, \mathbb{T}, \pi)$ is trajectoricaly dissipative.

Since dynamical system $(X, \mathbb{T}, \pi)$ satisfies conditions of Ladyzhenskaja then according to lemma 1.5 for any compact $M \in C(X)$ set $\Sigma^+(M)$ is relatively compact. According to lemma 1.7 dynamical system $(X, \mathbb{T}, \pi)$ is compactly dissipative. Theorem is proved.

Theorem 1.9. [21] *Let $(X, \mathbb{T}, \pi)$ is compactly k-dissipative and asymptotically compact, then $(X, \mathbb{T}, \pi)$ is locally k-dissipative.*

§ 2. Global attractors of nonautonomous disperse dynamical system.

Let $(Y, \mathbb{T}, \sigma)$ is dynamical system with uniqueness (i.e. $\pi(y, t)$ consist of unique point whatever are $y \in Y$ and $t \in \mathbb{T}$) and $(X, \mathbb{T}, \pi)$ is disperse dynamical system. Triple $< (X, \mathbb{T}, \pi), (Y, \mathbb{T}, \sigma), h >$ will be called nonautonomous disperse dynamical system where h is homomorphism $(X, \mathbb{T}, \pi)$ on $(Y, \mathbb{T}, \sigma)$ i.e. h is continuous mapping of X on Y satisfying condition : $h(\pi(x, t)) = \sigma(h(x), t)$ for all $x \in X$ and $t \in \mathbb{T}$.

Everywhere below (in this paragraph) we will suppose that Y is compact, (X, h, Y) is locally-trivial Banach fibering [24] and $|\cdot|$ is norm on (X, h, Y) compatible with metric ρ on X (i.e. $\rho(x_1, x_2) = | x_1 - x_2 |$ for any $x_1, x_2 \in X$ such that $h(x_1) = h(x_2)$).

Nonautonomous disperse dynamical system $< (X, \mathbb{T}, \pi), (Y, \mathbb{T}, \sigma), h >$ will be called pointwise (compactly, locally, boundedly) dissipative if such is (X, T, π).

We will call as a center of Levinson of nonautonomous dynamical system $< (X, \mathbb{T}, \pi), (Y, \mathbb{T}, \sigma), h) >$ a center of Levinson of $(X, \mathbb{T}, \pi)$.

Theorem 2.1. *Let $< (X, \mathbb{T}, \pi), (Y, \mathbb{T}, \sigma), h >$ is nonautonomous disperse dynamical system and let dynamical system $(X, \mathbb{T}, \pi)$ is completely continuous, i.e. for any $M \in B(X)$ exists $l = l(M) > 0$ such that $\pi^l M$ is relatively compact. Then following conditions are equivalent :*

1. there exists such positive number r, such that for any $x \in X$ and $\varphi_x \in \Phi_x$ will be found $\tau = \tau(x, \varphi_x) \geq 0$ for which $| \varphi_x(\tau) | < r$;

2. dynamical system $< (X, \mathbb{T}, \pi), (Y, \mathbb{T}, \sigma), h >$ is compactly dissipative and

$$\lim_{t \to +\infty} \sup\{\beta(x \cdot t, J) : | x | \leq R\} = 0 \tag{2.1}$$

for every $R > 0$, where J is center of Levinson of (X, T, π), i.e. nonautonomous disperse dynamical system $< (X, \mathbb{T}, \pi), (Y, \mathbb{T}, \sigma), h >$ admits compact global attractor.

Proof. Implication 2. $\Rightarrow$ 1. is evident. Will show that under conditions of theorem 2.1 the inverse implication takes place. Will set $A(r) = \{x \in X :| x |\leq r\}$, where $r > 0$ is number appearing in condition 1.. Since Y is compact and Banach fibering (X, h, Y) is locally-trivial, then is zero section $\Theta = \{\theta_y : y \in Y$where θ_y is zero element of fibre $X_y = h^{-1}(y)\}$ is compact and, consequently set $A(r)$ is bounded, because $A(r) \subseteq B[\Theta, r] = \{x \in X : \rho(x, \Theta) \leq r\}$. According to condition of theorem for bounded set $A(r)$ exists positive number l such that $\pi^l A(r)$ is relatively compact.

Let $x \in X, \varphi_x \in \Phi_x$ and $\tau = \tau(x, \varphi_x) \geq 0$ is such, that $\varphi_x(\tau) \in A(r)$, then $\varphi_x(\tau + l) \in K = \overline{\pi^l A(r)}$. Thus, dynamical system $(X, \mathbb{T}, \pi)$ is weakly dissipative and according to consequence 1.4 dynamical system $(X, \mathbb{T}, \pi)$ is compactly dissipative. Let J is center of Levinson $(X, \mathbb{T}, \pi)$ and $R > 0$ then set $A(R) = \{x \in X :| x |\leq R\}$, as it was mentioned above, is bounded and for it will be found number $l > 0$ such that $\pi^l A(R)$ is relatively compact. By virtue of compact dissipativity of $(X, \mathbb{T}, \pi)$ its center of Levinson J, according to theorem 1.1 attracts set $\pi^l A(R)$ and hence equality (2.1) takes place (2.1). Theorem is proved.

Theorem 2.2. *Let $< (X, \mathbb{T}, \pi), (Y, \mathbb{T}, \sigma), h >$ is nonautonomous disperse dynamical system and $(X, \mathbb{T}, \pi)$ satisfies condition of Ladyzhenskaya, then conditions 1. and 2. of theorem 2.1 are equivalent.*

Proof. Since Y is compact and vectorial fibering (X, h, Y) is locally trivial, then for any $R > 0$ set $\{x \in X :| x |\leq R\}$ is bounded. According to condition 1. of theorem 2.1 for any $x \in X$ $\varphi_x \in \Phi_x$ exists $\tau = \tau(x, \varphi_x) \geq 0$ such that $\varphi_x(\tau) \in A(r) = \{x \in X :| x |\leq r\}$. Now for the termination of theorem's demonstration is sufficient to refer on theorems 1.8 and 1.6 .

Dynamical systems $(X, \mathbb{T}, \pi)$ is called asymptotically compact it for any bounded closed positive invariant set M (i.e. $\pi^t M \subseteq M$ for all $t \geq 0$) exists non-empty compact such that holds equality (1.8).

Theorem 2.3. *Let $< (X, \mathbb{T}, \pi), (Y, \mathbb{T}, \sigma), h >$ is nonautonomous disperse dynamical system and $(X, \mathbb{T}, \pi)$ is asymptotically compact. Then following conditions are equivalent:*

1. there exists positive number R_0 and for any $R > 0$ will be found $l(R) > 0$ such that

$$| \pi^t x |\leq R_0 \tag{2.2}$$

for all $t \geq l(r)$ and $| x |\leq R$, where $| A |= \sup\{| a |: a \in A\}(A \subseteq X)$;

2. dynamical system $< (X, \mathbb{T}, \pi), (Y, \mathbb{T}, \sigma), h >$ admits compact global attractor.

Proof. It's evident that 1. follows from 2. That's why for theorem's demonstration will be sufficient to show, that 2. follows from 1. . Let $M_0 \subseteq A(R) = \{x \in X :| x |\leq R\}$. According to condition 1. for given number R will be found $l = l(R) > 0$ such that holds (2.2) and, particularly, set $M = \bigcup\{\pi^t M_0 : t \geq l(R_0)\}$ is bounded and positively invariant. In virtue of asymptotical compactness of (X, T, π) for set M will be found non-empty compact K for which takes place the equality (1.8). Now for the termination of theorem's demonstration is sufficient to refer on theorem 2.2. .Theorem is proved.

Theorem 2.4. *Let* $<(X,\mathbb{T},\pi),(Y,\mathbb{T},\sigma),h>$ *is nonautonomous disperse dynamical system and mappings* $\pi^t=\pi(\cdot,t):X\to 2^X(t\in\mathbb{T})$ *are presented in the form of sum* $\pi(x,t)=\varphi(x,t)+\psi(x,t)$ *for all* $t\in\mathbb{T}$ *and* $x\in X$ *and are fulfilled conditions :*

1.$|\varphi(x,t)|\le m(t,r)$ *for all* $t\in\mathbb{T}$ *and* $|x|\le r$, *where* $m:\mathbb{R}_+\times\mathbb{R}_+\to\mathbb{R}_+$ *and* $m(t,r)\to 0$ *when* $t\to+\infty$;

2. mappings $\psi(\cdot,t):X\to 2^X(t>0)$ *are conditionally completely continuous, i.e.* $\psi(A,t)$ *is relatively compact whatever are* $t>0$ *and bounded positively invariant set* A.

Then dynamical system $(X,\mathbb{T},\pi)$ *is asymptotically compact.*

Proof. Let $A\in\mathcal{B}(X)$ is such that $\Sigma^+(A)=\bigcup\{\pi^tA:t\ge 0\}$ is bounded, $r>0$ and $A\subseteq\{x\in X:|x|\le r\}$. Will show that whatever are $\{x_k\}$ and $t_k\to+\infty$ the sequence $\{y_k\}(y_k\in\pi(x_k,t_k))$ is relatively compact. Will convince ourselves that set $M=\{y_k\}$ may be covered by compact $\varepsilon-$net whatever is $\varepsilon>0$. Let $\varepsilon>0$ and $l>0$ are such, that $m(r,l)<\frac{\varepsilon}{2}$. Will present M in the form of union $M_1\bigcup M_2$ where $M_1=\{x_k\cdot t_k\}_{k=1}^{k_1}, M_2=\{x_k\cdot t_k\}_{k=k_1+1}^{\infty}$ and $k_1=\max\{k:t_k<l\}$. Set M_2 is subset of set $\pi^l(\Sigma^+(A))$ whose elements may be presented in the form $y_1+y_2(y_1\in\varphi(x,l),y_2\in\psi(x,l)$ and $x\in\Sigma^+(A))$. Since set $\psi(\Sigma^+(A),l)$ is relatively compact, then it may be covered by finite $\frac{\varepsilon}{2}-$ net. Will note, that for any $y\in\varphi(\Sigma^+(A),l)$ exists $x\in\Sigma^+(A)$ such that $y\in\varphi(x,l)$ and $|y|\le|\varphi(x,l)|\le m(l,r)<\frac{\varepsilon}{2}$, that's why zero section Θ of fibering (X,h,Y) is compact $\frac{\varepsilon}{2}$-not of set $\varphi(\Sigma^+(A),l)$.Thus M_2 and hence M is covered by $\frac{\varepsilon}{2}$-net and by virtue of completeness of space X set $M=\{y_k\}$ is relatively compact. Now termination of theorem's demonstration is sufficient to refer on lemma 1.5 . Theorem is proved.

Will remember that positive invariant compact set $M\subseteq Y$ is called minimal if it not contains own closed positive invariant subset.

The affirmations presented below revise theorems 2.1-2.3 in case, when Y is minimal.

Theorem 2.5. *Let* $<(X,\mathbb{T},\pi),(Y,\mathbb{T},\sigma),h>$ *is nonautonomous disperse dynamical system and are fulfilled following conditions:*

1. Y *is compact minimal set ;*

2. dynamical system $(X,\mathbb{T},\pi)$ *is completely continuous and all its positive semi-trajectories are bounded ;*

3. there exists $q\in Y$ *and* $r>0$ *such* $x\in X_q=h^{-1}(q)$ *and* $\varphi_x\in\Phi_x$ *will be found* $\tau=\tau(x,\varphi_x)\ge 0$ *for which* $|\varphi_x(\tau)|<r$.

Then dynamical system $<(X,\mathbb{T},\pi),(Y,\mathbb{T},\sigma),h>$ *admits compact global attractor.*

Proof. Will note that under conditions of theorem 2.5 if $r_0>r$, then for any $x\in X$ and $\varphi_x\in\Phi_x$ will be found $\tau=\tau(x,\varphi_x)\ge 0$ for which $|\varphi_x(\tau)|<r$. If we will assume that it's wrong, then will be found $r_0>r, q_0\in Y(q_0\ne q), x_0\in X_{q_0}$ and $\varphi_{x_0}\in\Phi_{x_0}$ such, that

$$|\varphi_{x_0}(\tau)|\ge\overline{r}_0>r \tag{2.3}$$

for all $\tau\ge 0$. By virtue of condition 2. set $\{\varphi_{x_0}(\tau):\tau\ge 0\}$ is relatively compact. Since set Y is minimal then $Y=\overline{\{\sigma(q_0,t):t\ge 0\}}$ therefore exists $t_n\to+\infty$ such that $\sigma(q_0,t_n)\to q$. By virtue of relatively compactness of $\{\varphi_{x_0}(\tau):\tau\ge 0\}$ consequence $\{\varphi_{x_0}(t_n)\}$ may be considered convergent. Will set $\overline{x}=\lim\limits_{n\to+\infty}\varphi_{x_0}(t_n)$, then according to theorem 0.1.1 [21] from consequence $\{\varphi_{x_0}(t+t_n)\}$ may be extracted subconsequence convergent uniformly on every compact from T to same motion $\varphi_{\overline{x}}$ of dynamical system (X,T,π). From (2.3) follows that

$$\mid \varphi_{\overline{x}}(t) \mid \geq \overline{r}_0 > r \tag{2.4}$$

for all $t \geq 0$. Inequality (2.4) contradicts to condition 3. of theorem. Obtained contradiction proves required affirmation. Now for termination of theorem's demonstration is sufficient to refer on theorem 2.1.

Theorem 2.6. *Let $<(X,\mathbb{T},\pi),(Y,\mathbb{T},\sigma),h>$ is nonautonomous disperse dynamical system and are fulfilled following conditions :*

1.Y is compact minimal set ;

2. dynamical system $(X,\mathbb{T},\pi)$ satisfies condition of Ladyzhenskaja ;

3. there exists $q \in Y$ and $r > 0$ such that for any $x \in X_q$ and $\varphi_x \in \Phi_x$ will be found $\tau = \tau(x,\varphi_x) \geq 0$ for which $\mid \varphi_(\tau) \mid < r$, then dynamical system $<(X,\mathbb{T},\pi),(Y,\mathbb{T},\sigma),h>$ admits compact global attractor.

Proof. Demonstration of formulated affirmation is based on the same scheme as for theorem 2.5 and uses theorem 2.2 .

§ 3. Global attractors of skew products of dynamical systems without uniqueness

Let $(Y,\mathbb{T},\sigma)$ is dynamical system with uniqueness on Y, W is complete metric space and φ is mapping of $T \times W \times Y$ in $C(W)(C(W)$ is family of all compacts in $W)$, satisfying following conditions :

1. $\varphi(0,u,y) = u$;
2. $\varphi(t+\tau,u,y) = \varphi(t,\varphi(\tau,u,y),\sigma(y,t))$ for all $t,\tau \in T, u \in W$ and $y \in Y$;
3. mapping $\varphi : \mathbb{T} \times W \times Y \to C(W)$ is $\beta-$ continuous.

Then triple $<W,\varphi,(Y,\mathbb{T},\sigma)>$ will be called skew product (without uniqueness) over $(Y,\mathbb{T},\sigma)$ with fibre W.

Will set $X = W \times Y$ and will define mapping $\pi : X \times \mathbb{T} \to C(X)$ by the following rule : $\pi = (\varphi,\sigma)$ i.e. $\pi(<u,y>,t) =<\varphi(t,u,y),\sigma(y,t)>= \bigcup\{<w,\sigma(y,t)>: w \in \varphi(t,u,y)\}$ then it may be verified that $(X,\mathbb{T},\pi)$ is disperse dynamical system on X and mapping $h = pr_2 : X \to Y$(i.e. $h(u,y) = y$ for all $u \in W$ and $y \in Y$) is continuous and satisfies condition :

4. $h(\pi(x,t)) = \sigma(h(x),t)$ for all $x \in X$ and $t \in \mathbb{T}$, i.e. h is homomorphism of $(X,\mathbb{T},\pi)$ on (Y,T,σ) and, hence, triple $<(X,\mathbb{T},\pi),(Y,\mathbb{T},\sigma),h>$ is nonautonomous disperse dynamical system.

Thus, using skew product (without uniqueness) $<W,\varphi,(Y,\mathbb{T},\sigma)>$ of dynamical system $(Y,\mathbb{T},\sigma)$ with fibre W in natural way is constructed nonautonomous disperse dynamical system $<(X,\mathbb{T},\pi),(Y,\mathbb{T},\sigma),h>$ $(X = W \times Y, \pi = (\varphi,\sigma)$ and $h = pr_2 : X \to Y)$, which we will call nonautonomous disperse dynamical system, associated by skew product $<W,\varphi,(Y,\mathbb{T},\sigma)>$ over $(Y,\mathbb{T},\sigma)$ with fibre W.

If $M \subseteq W$, then will set

$$\Omega_y(M) = \bigcap_{t\geq 0} \overline{\bigcup_{\tau \geq t} U(\tau,y^{-\tau})M} \tag{3.1}$$

for every $y \in Y$, where $y^{-\tau} = \sigma(-\tau,y)$ and $U(t,y) = \varphi(t,\cdot,y)$.

Lemma 3.1. *Following affirmations take place:*

1. point $w \in \Omega_y(M)$ if and only if, there exists $t_n \to +\infty, \{x_n\} \subseteq M$ and $w_n \in U(t_n, y^{-t_n})x_n$ such that $w = \lim_{n\to+\infty} w_n$;

2. $U(t,y)\Omega_y(M) \subseteq \Omega_{yt}(M)$ for all $y \in Y$ and $t \in \mathbb{T}_+$;

3. if there exists non-empty compact $K \subset W$ such that

$$\lim_{t\to+\infty} \beta(\varphi(t, M, y^{-t}), K) = 0, \tag{3.2}$$

then $\Omega_y(M) \neq \emptyset$, is compact,

$$\lim_{t\to+\infty} \beta(\varphi(t, M, y^{-t}), \Omega_y(M)) = 0, \tag{3.3}$$

and

$$U(t,y)\Omega_y(M) = \Omega_{yt}(M) \tag{3.4}$$

for all $y \in Y$ and $t \in \mathbb{T}_+$.

Proof. First affirmation of lemma follows directly from equality (3.1). The second affirmation of lemma follows from definition of set $U(t,y)\Omega_y(M), \Omega_{yt}(M)$, from equality $U(t,y)U(\tau, y^\tau) = U(t+\tau, (y^t)^{-t-\tau})$ for all $t, \tau \geq 0, y \in Y$ and $\beta-$ continuity of mapping $\varphi : \mathbb{T}_+ \times W \to C(W)$. Really,

$$U(t,y)\Omega_y(M) = U(t,y)(\bigcap_{s\geq 0} \overline{\bigcup_{\tau\geq s} U(\tau, y^{-\tau})M})$$

$$\subseteq \bigcap_{s\geq 0} U(t,y)(\overline{\bigcup_{\tau\geq s} U(\tau, y^{-\tau})M}) \subseteq \bigcap_{s\geq 0} \overline{\bigcup_{\tau\geq s} U(t,y)U(\tau, y^{-\tau})M} =$$

$$\bigcap_{s\geq 0} \overline{\bigcup_{\tau\geq s} U(t+\tau, (yt)^{-t-\tau})M} \subseteq \bigcap_{s\geq 0} \overline{\bigcup_{\tau\geq s} U(\tau, (yt)^{-\tau})M} = \Omega_{yt}(M).$$

Equality (3.3) follows directly from the first affirmation of lemma and equality (3.2).

Will show that takes place equality (3.4). It's sufficient to show that $\Omega_{yt}(M) \subseteq U(t,y)\Omega_y(M)$ for all $y \in Y$ and $t \geq 0$. Let $y \in Y, t \geq 0$ and $w \in \Omega_{yt}(M)$, then according to first affirmation of lemma exists $x_n \in M, t_n \to +\infty$ and $w_n \in U(t_n, y^{t-t_n})x_n$ such, that $w = \lim_{n\to+\infty} w_n$. Since $U(t,y)U(t_n - t, y^{t-t_n})x_n$ for sufficiently big $n(t_n \geq t)$, then exist $\overline{w}_n \in U(t_n - t, y^{t-t_n})x_n$ such that $w_n \in U(t,y)\overline{w}_n$. Under conditions of lemma 3.1 sequence $\{\overline{w}_n\}$ may be considered convergent. Let $\overline{w} = \lim_{n\to+\infty} \overline{w}_n$, then according to first affirmation of lemma $\overline{w} \in \Omega_y(M)$ and, consequently, $w \in U(t,y)\Omega_y(M)$, i.e. $\Omega_{yt}(M) \subseteq U(t,y)\Omega_y(M)$. Lemma is proved.

Will call skew product over $(Y, \mathbb{T}, \sigma)$ with fibre W as a compactly dissipative, if there exists non-empty compact $K \subseteq W$ such, that

$$\lim_{t\to+\infty} \sup\{\beta(U(t,y)M, K) : y \in Y\} = 0 \tag{3.5}$$

for all $M \in C(W)$.

Lemma 3.2. *Let Y is compact and $< W, \varphi, (Y, \mathbb{T}, \sigma) >$, is skew product (without uniqueness) over $(Y, \mathbb{T}, \sigma)$ with fibre W. In order that $< W, \varphi, (Y, \mathbb{T}, \sigma) >$, will be compactly dissipative is necessary and sufficient that semigroup autonomous system $(X, \mathbb{T}_+, \pi)(X = W \times Y, \pi = (\varphi, \sigma))$ to be compactly dissipative.*

The formulated affirmation directly follows from respective definitions.

Will say that space X has property (S), if for any compact $K \subseteq X$ exists connected set $M \subseteq X$ such that $K \subseteq M$.

By entire trajectory of semi-group dynamical $(X, \mathbb{T}_+, \pi)$ (of skew product $< W, \varphi, (Y, \mathbb{T}, \sigma) >$ over $(Y, \mathbb{T}, \sigma)$ with fibre W), passing through point $x \in X$ ($(u, y) \in W \times Y$) is called continuous mapping $\gamma : \mathbb{T} \to X(\nu : \mathbb{T} \to W)$ satisfying conditions : $\gamma(0) = x(\nu(0) = w)$ and $\gamma(t + \tau) \in \pi^t\gamma(\tau)(\gamma(t + \tau) \in \varphi(t, \nu(\tau), y\tau))$ for all $t \in \mathbb{T}_+$ and $\tau \in \mathbb{T}$.

Theorem 3.3. *Let Y is compact, $< W, \varphi, (Y, \mathbb{T}, \sigma) >$ is compactly dissipative and K is non-empty compact, appearing in equality (3.5), then:*

1. $I_y = \Omega_y(K) \neq \emptyset$, is compact, $I_y \subseteq K$ and

$$\lim_{t\to+\infty} \beta(U(t, y)K, I_y) = 0 \tag{3.6}$$

for every $y \in Y$;

2. $U(t, y)I_y = I_{yt}$ for all $y \in Y$ and $t \in \mathbb{T}_+$;

3.

$$\lim_{t\to+\infty} \beta(U(t, y)M, I_y) = 0 \tag{3.7}$$

for all $M \in C(W)$ and $y \in Y$;

4.

$$\lim_{t\to+\infty} \sup\{\beta(U(t, y)M, I) : y \in Y\} = 0 \tag{3.8}$$

whatever is $M \in C(W)$, where $I = \bigcup\{I_y : y \in Y\}$;

5. $I = pr_1J$ and $I_y = pr_1J_y$, where J is center of Levinson of (X, T_+, π) and $J_y = J \bigcap X_y$;

6. set I is compact ;

7. set I is connected if is fulfilled one of the following two conditions :

a. $\mathbb{T}_+ = \mathbb{R}_+$ and spaces W and Y are connected ;

b. $\mathbb{T}_+ = \mathbb{Z}_+$ and space $W \times Y$ has property (S) or is connected and locally connected.

Proof. First two affirmations of theorem follow from lemma 3.2.

If we will allow that equality (3.7) does not take place, then will be found $\varepsilon_0 > 0, y_0 \in Y, M_0 \in C(W), t_n \to +\infty$ and $w_n \in U(t_n, y_0^{-t_n})M_0$ such, that

$$\rho(w_n, I_{y_0}) \geq \varepsilon_0 \tag{3.9}.$$

According to (3.6) for ε_0 and $y_0 \in Y$ will be found $t_0 = t_0(\varepsilon_0, y_0) > 0$ such, that

$$\beta(U(t, y^{-t})K, I_{y_0}) < \frac{\varepsilon_0}{2} \tag{3.10}$$

for all $t \geq t_0$. Will notice that

$$U(t_n, y_0^{-t_n}) = U(t_0, y_0^{-t_0})U(t_n - t_0, y_0^{-t_n}). \tag{3.11}$$

Let $\overline{w}_n \in U(t_n - t_0, y_0^{-t_n})M_0$ such, that $w_n \in U(t_0, y_0^{-t_0})\overline{w}_n$. By virtue of compact dissipativity $< W, \varphi, (Y, \mathbb{T}, \sigma) >$ sequences $\{w_n\}$ and $\{\overline{w}_n\}$ may be considered convergent. Will set $\overline{w} = \lim\limits_{n\to+\infty} \overline{w}_n$ and $w = \lim\limits_{n\to+\infty} w_n$, then $w \in U(t_0, y_0^{-t_0})\overline{w}$ and according to (3.5) $\overline{w} \in K$. Passing to limit in (3.9), when $n \to +\infty$ and taking into account (3.11) we will receive

$$\rho(w, I_{y_0}) \geq \varepsilon_0. \tag{3.12}$$

On the other hand , since $\overline{w} \in K$, then from (3.10) we have $U(t_0, y_0^{-t_0})\overline{w} \subseteq B(I_{y_0}, \frac{\varepsilon_0}{2})$ and, consequently

$$w \in U(t_0, y_0^{-t_0})\overline{w} \subseteq B(I_{y_0}, \frac{\varepsilon_0}{2}), \tag{3.13}$$

which contradicts to (3.12). Obtained contradiction proves required affirmation.

Now we will prove equality (3.8). If we will allow, that it does not take place then will be found $\varepsilon_0 > 0, M_0 \in C(W), y_n \in Y, \{x_n\} \subseteq M_0, t_n \to +\infty$ and $w_n \in U(t_n, y_n^{-t_n})x_n$ such, that

$$\rho(w, I) \geq \varepsilon_0. \tag{3.14}$$

By virtue of compactness of Y sequences $\{y_n\}$ and $\{y_n t_n\}$ may be considered convergent. Will set $y_0 = \lim\limits_{n\to+\infty} y_n$ and $\overline{y} = \lim\limits_{n\to+\infty} y_n t_n$. According to 3.7) for number $\varepsilon_0 > 0$ and $y_0 \in Y$ will be found $t_0 = t_0(\varepsilon_0, y_0) > 0$ such, that takes place (3.10) for all $t \geq t_0(\varepsilon_0, y_0)$. By virtue of compactness M_0 and compact dissipativity $< W, \varphi, (Y, \mathbb{T}, \sigma) >$ sequences $\{\overline{w}_n\}$ and w_n may be considered convergent, where $\{\overline{w}_n\} \in U(t_n - t_0, y_n^{-t_n})x_n$ and $w_n \in U(t_0, y_n^{-t_0})\{\overline{w}_n\}$. Will notice, that according to (3.5) $\overline{w} \in K$. From equality (3.11) follows, that $w \in U(t_0, y_0^{-t_0})\overline{w}$ and, consequently, from (3.14) we have

$$w \notin B(I_{y_0}, \frac{\varepsilon_0}{2}). \tag{3.15}$$

Relation (3.15) contradicts to (3.10), which completes the proof of fourth affirmation of theorem.

Will prove the fifth affirmation of theorem. Will notice, that $w \in I_y$, if through the point $(w, y) \in W \times Y$ passes such an entire trajectory ν of skew product $< W, \varphi, (Y, \mathbb{T}, \sigma) >$ that $\nu(\mathbb{T})$ is relatively compact. Really, since $w \in \varphi(t, \nu(-t), y^{-t})$ for all $t \in \mathbb{T}$, then from equality (3.7) follows necessary inclusion. Now it remains to notice, that center of Levinson J is compact and consists of entire trajectories $(X, \mathbb{T}_+, \pi)$ and, hence, $pr_1 J \subseteq I$ and $pr_1 J_y \subseteq I_y$ for all $y \in Y$. Will set $M_y = I_y \times \{y\}$ and $M = \bigcup\{M_y : y \in Y\}$, then from the second affirmation of theorem follows invariance of M. It's evident that M is compact and, consequently, $M \subseteq J$ because J is a maximal compact invariant set in $(X, \mathbb{T}_+, \pi)$. Therefore $I_y \subseteq pr_1 J_y$ for all $y \in Y$ and, consequently, $I \subseteq pr_1 J$. Thus $pr_1 J_y = I_y$ for all $y \in Y$ and $pr_1 J = I$.

Compactness of set I follows from equality $I = pr_1 J$, compactness of J and continuity of $pr_1 : X \to W$.

Last affirmation of theorem follows from that fact that under conditions of theorem 3.3 center of Levinson J of dynamical system $(X, \mathbb{T}_+, \pi)$ is connected according to consequence 1.8.7 and theorem 1.8.15 from [21], and consequently, I is also connected as a continuous image of connected set . Theorem is proved.

Observation. . *Affirmations 1.-4. and 6. for finite-dimensional discrete systems are proved in [25].*

§ 4. Global attractors of differential and difference inclusions.

Will show some examples of skew products of dynamical systems playing an important role in the study of differential, difference, functionally-differential and evolutionary equations and inclusions.

Example 4.1. (Ordinary differential equations without uniqueness). Let E^n is $n-$ dimensional real or complex Euclidean space with norm $|\cdot|$. Will note by $C(\mathbb{R}\times E^n, E^n)$ a set of all continuous mappings $f:\mathbb{R}\times E^n\to E^n$ allotted with uniform convergence topology on compacts from $\mathbb{R}\times E^n$. Will examine differential equation

$$u' = f(t,u), \tag{4.1}$$

where $f\in C(\mathbb{R}\times E^n, E^n)$. Parallel with equation (4.1) will examine and family of differential equations

$$v' = g(t,v), \tag{4.2}$$

where $g\in H(f)=\overline{\{f_\tau : \tau\in R\}}$, f_τ is translation on τ by variable t of function f and by stroke is noted closer in $C(\mathbb{R}\times E^n, E^n)$.

Will call function $f\in C(\mathbb{R}\times E^n, E^n)$ regular, if for any $v\in E^n$ and $g\in H(f)$ equation (4.2) admits at least one solution $\varphi_{(v,g)}(t)$ defined on R_+ and passing through point v when $t=0$. Will note by $\Phi_{(v,g)}$ a set of all solutions of equation (4.2), defined on R_+ and passing through point v when $t=0$.

Let $f\in C(\mathbb{R}\times E^n, E^n)$ is regular, will set $\varphi(t,v,g)=\{\varphi_{(v,g)}(t):\varphi_{(v,g)}\in\Phi_{(v,g)}\}$, then $\varphi:\mathbb{R}_+\times E^n\times H(f)\to C(E^n)$ and from general properties of solutions of differential equations [26] follows, that are fulfilled following conditions :

a. $\varphi(0,v,g)=v$ for all $v\in E^n, g\in H(f)$;

b. $\varphi(t+\tau,v,g)=\varphi(t,\varphi(\tau,v,g),g_\tau)$ for all $v\in E^n, g\in H(f)$ and $t,\tau\in\mathbb{R}_+$;

c. mapping $\varphi:\mathbb{R}_+\times E^n\times H(f)\to C(E^n)$ is $\beta-$ continuous.

Will note by $Y=H(f)$ and $(Y,\mathbb{R},\sigma)$ a dynamical system of translations on Y,then triplet $< E^n,\varphi,(Y,\mathbb{R},\sigma) >$ is a skew product (without uniqueness) of $(Y,\mathbb{R},\sigma)$ with fibre E^n. Thus differential equation (4.1) with regular right part $f\in C(\mathbb{R}\times E^n, E^n)$ naturally generates nonautonomous disperse dynamical system $< (X,\mathbb{R}_+,\pi),(Y,\mathbb{R},\sigma),h) >$, where $X=E^n\times Y, \pi=(\varphi,\sigma)$ and $h=pr_2:X\to Y$. Applying to constructed dynamical system general results from paragraph 1.-3. we will obtain a series of affirmations concerning to equation (4.1). Below are presented some affirmations of such kind.

Theorem 4.1. *Let $H(f)$ is compact and $f\in C(\mathbb{R}\times E^n, E^n)$ is regular then following conditions are equivalent :*

a. exists $r>0$ such that whatever are $v\in E^n, g\in H(f)$ and $\varphi_{(v,g)}\in\Phi_{(v,g)}$ there will be found number $\tau>0$ such that $|\varphi_{(v,g)}(\tau)|<r$;

b. exists $R>0$ such that $\lim\limits_{t\to+\infty}|\varphi_{(v,g)}(t)|<R$ for all $(v,g)\in E^n\times H(f)$ and $\varphi_{(v,g)}\in\Phi_{(v,g)}$;

c. exists $R_0>0$ such that for any $R>0$ will be found $L(R)>0$ such that $|\varphi(t,v,g)|\le R_0$ for all $t\ge L(R), |v|\le R$ and $g\in H(f)$.

This affirmation strengthens one results from [27].

Will say that set $I\subset E^n$ is invariant relatively to nonautonomous equation (4.1) for any $u\in I$ exists $g\in H(f)$ and solution $\varphi_{(v,g)}$ of equation (4.2) defined on R such that $\varphi_{(v,g)}(t)\in I$ for all $t\in\mathbb{R}$.

Will note by I_g set of all points from I through which passes at least one solution of equation (4.2) defined on R and dying entirely in I. Then $I = \bigcup\{I_g : g \in H(f)\}$.

Theorem 4.2. *Let $H(f)$ is compact and $f \in C(\mathbb{R} \times E^n, E^n)$ is regular, then following conditions are equivalent :*

a. equation (1) is dissipative, i.e. is fulfilled one of three conditions a.-b. from theorem 4.1. ;

b. equation (1) admits compact global attractor, i.e. exists non-empty compact invariant relatively to (4.1) set I such that :

b.1. for any $\varepsilon > 0$ exists $\delta(\varepsilon) > 0$ such that $\rho(v, I_g) < \delta$ implies $\beta(\varphi(t, v, g), I_{g_t}) < \varepsilon$;

b.2. takes place the equality

$$\lim_{t \to +\infty} \sup_{|v| \leq R, g \in H(f)} \beta(\varphi(t, v, g), I) = 0$$

for every $R > 0$.

Theorem 4.3. *Let $f \in C(\mathbb{R} \times E^n, E^n)$ is regular and recurrent by variable $t \in \mathbb{R}$ uniformly by x on compacts from E^n, i.e. $H(f)$ is compact and $H(f) = H(g)$ for all $g \in H(f)$. The following conditions are equivalent :*

a. all solutions of all equations of family (4.2) are bounded on $\mathbb{R}_+$ and for any $v \in E^n$ and $\varphi_{(v,g)} \in \Phi_{(v,g)}$ exists $\tau > 0$ such that $| \varphi_{(v,f)}(\tau) | < r$;

b. exists $R_0 > 0$ such that for any $R > 0$ will be found $L(R) > 0$ such that $| \varphi(t, v, g) | \leq R_0$ for all $t \geq L(R), | v | \leq R$ and $g \in H(f)$.

The formulated affirmation generalizes theorem 1.6.2 from [28] .

Example 4.2. (Differential inclusions). Will note by $C_V(E^n)$ family of all convex compacts from E^n, and by $C(\mathbb{R} \times E^n, C_V(E^n))$ set of all continuous in Hausdorff's metric [23,29,30] mappings $F : \mathbb{R} \times E^n \to C_V(E^n)$, allotted by uniform convergence topology on compacts. Will examine differential inclusion

$$u' \in F(t, u), \tag{4.3}$$

where $f \in C(\mathbb{R} \times E^n, C_V(E^n))$. Parallel with inclusions (4.3) will also examine and family of differential inclusions

$$v' \in G(t, v), \qquad (4,4)$$

where $G \in H(F) = \overline{\{F_\tau : \tau \in \mathbb{R}\}}, F_\tau$ is translation by variable t of function F on τ and by stroke is noted closure inn $C(\mathbb{R} \times E^n, C_V(E^n))$.

Will call function $F \in C(\mathbb{R} \times E^n, C_V(E^n))$ regular, if for every inclusion (4.4) is fulfilled condition of existence and non-local extendibility to the right, i.e. for any $G \in H(F)$ and $v \in E^n$ exists at least one solution $\varphi_{(v,G)}(t)$ of inclusion (4.4) passing through point v when $t = 0$ and all solutions are defined on $\mathbb{R}_+$.

Let $F \in C(\mathbb{R} \times E^n, C_V(E^n))$ is regular. Will set $\varphi(t, v, g) = \{\varphi_{(v,G)}(t) : \varphi_{(v,G)} \in \Phi_{(v,G)}\}$, where $\Phi_{(v,G)}$ is set of all solutions of inclusion (4.4) defined on $\mathbb{R}_+$ and passing through point v when $t = 0$. From general properties of differential inclusions [29] follows, that following properties take place :

a. $\varphi(0, v, G) = v$ for all $v \in E^n, G \in H(F)$;

b. $\varphi(t + \tau, v, G) = \varphi(t, \varphi(\tau, v, G), G_\tau)$ for all $v \in E^n, G \in H(F)$ and $t, \tau \in \mathbb{R}_+$;

c. mapping $\varphi : \mathbb{R}_+ \times E^n \times H(F) \to C(E^n)$ is $\beta-$ continuous.

Will note by $Y = H(F)$ and $(Y, \mathbb{R}, \sigma)$ a dynamical system of translations on Y,then triplet $< E^n, \varphi, (Y, \mathbb{R}, \sigma) >$ is a skew product (without uniqueness) of $(Y, \mathbb{R}, \sigma)$ with fibre E^n. Thus differential inclusion (4.3) with regular right part $F \in C(\mathbb{R} \times E^n, C_V(E^n))$ naturally generates nonautonomous disperse dynamical system $< (X, \mathbb{R}_+, \pi), (Y, \mathbb{R}, \sigma), h >$, where $X = E^n \times Y, \pi = (\varphi, \sigma)$ and $h = pr_2 : X \to Y$.

Applying to constructed dynamical system general results from paragraph 1.-3. we will obtain following results.

Theorem 4.4. *Let $F \in C(\mathbb{R}\times E^n, C_V(E^n)), H(F)$ is compact in $C(\mathbb{R}\times E^n, C_V(E^n))$ and F is regular, then following conditions are equivalent :*

a. exists $r > 0$ such that whatever are $v \in E^n, G \in H(G)$ and $\varphi_{(v,G)} \in \Phi_{(v,G)}$ there will be found number $\tau > 0$ such that $| \varphi_{(v,G)}(\tau) |< r$;

b. exists $R_0 > 0$ such that for any $R > 0$ will be found $L(R) > 0$ such that $| \varphi(t, v, G) |\leq R_0$ for all $t \geq L(R), | v |\leq R$ and $G \in H(F)$.

c. exists $R > 0$ such that $\overline{\lim}_{t\to+\infty} | \varphi_{(v,G)}(t) |< R$ for all $(v, G) \in E^n \times H(F)$ and $\varphi_{(v,G)} \in \Phi_{(v,G)}$.

Will call differential inclusion (4.3) dissipative, if is fulfilled at least one of conditions of theorem 4.3.

Will call set $I \subset E^n$ invariant relatively to differential inclusion (4.3) if for any $v \in I$ exists $G \in H(F)$ and solution $\varphi_{(v,G)}$ of differential inclusion (4.4) defined on $\mathbb{R}$ such that $\varphi_{(v,G)}(t) \in I$ for all $t \in \mathbb{R}$.

Will note by I_G set of all points from I through which passes at least one solution of differential inclusion (4.4) defined and lying in I , then $I = \bigcup\{I_G : G \in H(F)\}$.

Non-empty , compact, and invariant relatively to differential inclusion (4.3) set $I \subset E^n$ will be called global attractor for (4.3), if are fulfilled following conditions :

1. I is orbitally stable, i.e. for all $\varepsilon > 0$ exists $\delta(\varepsilon) > 0$ such that $\rho(v, I_G) < \delta$ implies $\beta(\varphi(t, v, G), I_G) < \varepsilon$ for all $t \geq 0$;

2. takes place equality

$$\lim_{t\to+\infty} \sup_{|v|\leq R, G\in H(F)} \beta(\varphi(t, v, G), I) = 0$$

for every $R > 0$.

Theorem 4.5. *Let $F \in C(R\times E^n, C_V(E^n)), H(F)$ is compact in $C(\mathbb{R}\times E^n, C_V(E^n))$ and F is regular, then following condition are equivalent :*

a. differential inclusion (4.3) is dissipative ;

b. differential inclusion (4.3) admits compact global attractor.

Theorem 4.6. *Let $F \in C(\mathbb{R} \times E^n, C_V(E^n))$ is regular and recurrent by variable $t \in \mathbb{R}$ uniformly by x on compacts from E^n. Then following conditions are equivalent :*

a. all solutions of all inclusions of family (4.4) are bounded on $\mathbb{R}_+$ and for any $v \in E^n$ and $\varphi_{(v,F)} \in \Phi_{(v,F)}$ exists $\tau > 0$ such that $| \varphi_{(v,F)}(\tau) |< r$;

b. exists $R_0 > 0$ such that for any $R > 0$ will be found $L(R) > 0$ such that $| \varphi(t, v, G) |\leq R_0$ for all $t \geq L(R), | v |\leq R$ and $G \in H(F)$.

Observation. . *If I is compact global attractor of equation (4.1) (differential inclusion (4.3)) and $g \in H(f)(G \in H(F))$, then set $I_g(I_G)$ consists exactly from those points of E^n through which passes at least one solution of equation (4.2) (differential inclusion (4.4)) defined and bounded on R.*

Example 4.3. (Difference inclusions). Will examine difference inclusion

$$u_{t+1} \in F(t, u_t), \tag{4.5}$$

where $F \in C(\mathbb{Z} \times E^n, C(E^n))$. Parallel with difference inclusion (4.5) will examine family of difference inclusions

$$v_{t+1} \in G(t, v_t), \tag{4.6}$$

where $G \in H(F) = \overline{\{F_\tau : \tau \in \mathbb{Z}\}}, F_\tau(t, u) = F(t+\tau, u)$ and by stroke is note closure in $C(\mathbb{Z} \times E^n, C(E^n))$.

Will note by $\varphi_{(v,G)}(n)$ solution of inclusion (4.6) passing through point v for $t = 0$ end defined for all $t \geq 0$. Will set $\varphi(t, v, G) = \{\varphi_{(v,G)}(t) : \varphi_{(v,G)} \in \Phi_{(v,G)}\}$, where $\Phi_{(v,G)}$ is set of all solutions of inclusion (4.6) , passing through point v for $t = 0$. From general properties of difference inclusions (see for ex. [25]) follows that mapping $\varphi : \mathbb{Z}_+ \times E^n \times H(F) \to C(E^n)$ possesses following properties :

1. $\varphi(0, v, G) = v$ for all $v \in E^n, G \in H(F)$;
2. $\varphi(t+\tau, v, G) = \varphi(t, \varphi(\tau, v, G), G_\tau)$ for all $v \in E^n, G \in H(F)$ and $t, \tau \in \mathbb{Z}_+$;
3. mapping $\varphi : \mathbb{Z}_+ \times E^n \times H(F) \to C(E^n)$ is $\beta-$ continuous.

Will set $Y = H(F)$ and will note by $(Y, \mathbb{T}_+, \sigma)$ disperse dynamical system of translations on Y, then triple $< E^n, \varphi, (Y, \mathbb{Z}_+, \sigma) >$ is skew product (without uniqueness) of $(Y, \mathbb{R}_+, \sigma)$ with fibre E^n. Thus, nonautonomous difference inclusion (4.5) is natural way generates nonautonomous disperse dynamical system $< (X, \mathbb{Z}_+, \pi), (Y, \mathbb{Z}_+, \sigma), h >$, where $X = E^n \times Y, \pi = (\varphi, \sigma)$ and $h = pr_2 : X \to Y$. Applying results paragraph 1-3 to constructed above nonautonomous dynamical system will obtain following results.

Theorem 4.7. *Let $H(F)$ is compact in $C(\mathbb{Z} \times E^n, C(E^n))$, then following affirmations are equivalent :*

a. exists $r > 0$ such that whatever are $v \in E^n, G \in H(F)$ and $\varphi_{(v,G)} \in \Phi_{(v,G)}$ there will be found $k \in \mathbb{Z}_+$ such that $\dim \varphi_{(v,G)}(\tau) \dim < r$;

b. exists $R_0 > 0$ such, that for any $R > 0$ will be found $L(R) > 0$ such that $| \varphi(t, v, G) | \leq R_0$ for all $t \geq L(R), | v | \leq R$ and $G \in H(F)$;

c. exists $R > 0$ such that $\overline{\lim}_{t \to +\infty} | \varphi_{(v,G)}(t) | < R$ for all $(v, G) \in E^n \times H(F)$ and $\varphi_{(v,G)} \in \Phi_{(v,G)}$;

Will call difference inclusion (4.5) dissipative if is fulfilled at least one of conditions a.-c. of theorem 4.7.

Will call set $I \subset E^n$ invariant relatively to difference inclusion (4.5) , if for any $v \in I$ exists $G \in H(F)$ and solution $\varphi_{(v,G)}$ of difference inclusion (4.6) defined on $\mathbb{Z}$ such, that $\varphi_{(v,G)}(t) \in I$ for all $t \in \mathbb{Z}$.

Will note by I_G set of all points from I through which passes at least one solution of difference inclusion (4.6) defined on R and lying entirely in I, then $I = \bigcup\{I_G : G \in H(F)\}$.

Non-empty, compact, and invariant relatively to difference inclusion (4.5) set $I \subset E^n$ will be called global attractor for (4.5), if it is orbitally asymptotical stable on the whole, i.e. are fulfilled following conditions :

1. for any $\varepsilon > 0$ exists $\delta(\varepsilon) > 0$ such, that $\rho(v, I_G) < \delta$ implies $\beta(\varphi(t, v, G), I_{G_t}) < \varepsilon$ for all $t \in \mathbb{Z}_+$;
2. takes place equality

$$\lim_{t \to +\infty} \sup_{|v| \leq R, G \in H(F)} \beta(\varphi(t, v, G), I) = 0$$

for every $R > 0$.

Theorem 4.8. *Let $H(F)$ is compact in $C(Z \times E^n, C(E^n))$, then following affirmations are equivalent :*

a. difference inclusion (4.5) is dissipative ;

b. difference inclusion (4.5) admits compact global attractor.

Affirmation close to theorem 4.8 is contained in work [25].

Theorem 4.9. *Let $C(\mathbb{Z} \times E^n, C(E^n))$ is recurrent by variable $t \in \mathbb{Z}$, uniformly by x on compacts from E^n. Then following conditions are equivalent :*

a. all solutions of all inclusions of family (4.6) are bounded on $\mathbb{Z}_+$ and for any $v \in E^n$ and $\varphi_{(v,F)} \in \Phi_{(v,F)}$ exists $\tau > 0$ such, that $| \varphi_{(v,G)}(\tau) | < r$;

b. exists $R_0 > 0$ such, that for any $R > 0$ will be found $L(R) > 0$ such, that $| \varphi(t, v, G) | \leq R_0$ for all $t \geq L(R), | v | \leq R$ and $G \in H(F)$.

Example 4.4. (Functionally-differential equations without uniqueness).

Let $r > 0, C([a, b], \mathbb{R}^n)$ is Banach space of continuous functions $\nu : [a, b] \to R^n$ with norm sup. If $[a, b] = [-r, 0]$, then will set $C = C([-r, 0], \mathbb{R}^n)$. Let $\sigma \in R, A \geq 0$ and $u \in C([\sigma - r, \sigma + A], \mathbb{R}^n)$. For any $t \in [\sigma, \sigma + A]$ will define $u_t \in C$ by relation $u_t(\theta) = u(t + \theta), -r \leq \theta \leq 0$. Will note by $\mathbb{P}(\mathbb{R} \times C, \mathbb{R}^n)$ a space of all continuous functions $f : R \times C \to \mathbb{R}^n$,bounded on the bounded subsets from $\mathbb{R} \times C$ and continuous with respect to $t \in \mathbb{R}$ uniformly by x on bounded subsets from C . Will provide $\mathbb{P}(\mathbb{R} \times C, \mathbb{R}^n)$ with topology of uniform convergence on bounded subsets from $\mathbb{R} \times C$. This topology may be given with the help of metric

$$d(f, g) = \sum_{n=1}^{\infty} \frac{1}{2^n} \frac{d_n(f, g)}{1 + d_n(f, g)}, \tag{4.7}$$

where

$$d_n(f, g) = \sup_{|t| \leq n, ||\varphi|| \leq n} | f(t, \varphi) - g(t, \varphi) | \ (n \in \mathbb{N}).$$

Will note, that set set $\mathbb{P}(\mathbb{R} \times C, \mathbb{R}^n)$, allotted by metric (4.7), turns into metric space, invariant relative to shifts by $t \in \mathbb{R}$. Will note by $f_\tau(\tau)$ a shift of function $f \in \mathbb{P}(\mathbb{R} \times C, \mathbb{R}^n)$ on τ with respect to $t \in \mathbb{R}$, i.e. $f_\tau(t, \varphi) = f(t + \tau, \varphi)((t, \varphi) \in \mathbb{R} \times C)$. Will define mapping $\sigma : \mathbb{P}(\mathbb{R} \times C, \mathbb{R}^n) \times \mathbb{R} \to \mathbb{P}(\mathbb{R} \times C, \mathbb{R}^n)$ by the following rule : $\sigma(f, \tau) = f_\tau$ for all $(f, \tau) \in \mathbb{P}(\mathbb{R} \times C, \mathbb{R}^n) \times \mathbb{R}$. It easily can be verified that $\sigma(f, 0) = f$ and $\sigma(\sigma(f, \tau_1), \tau_2) = \sigma(f, \tau_1 + \tau_2)$ for all $f \in \mathbb{P}(\mathbb{R} \times C, \mathbb{R}^n)$ and $\tau_1, \tau_2 \in \mathbb{R}$. As well as in lemma 1.0.1 [21] can be verified continuity of mapping $\sigma : \mathbb{P}(\mathbb{R} \times C, \mathbb{R}^n) \times \mathbb{R} \to$

$\mathbb{P}(\mathbb{R}\times C,\mathbb{R}^n)$ and, consequently, triple $(\mathbb{P}(\mathbb{R}\times C,\mathbb{R}^n),\mathbb{R},\sigma)$ is dynamical system of shifts on $\mathbb{P}(\mathbb{R}\times C,\mathbb{R}^n)$.

Will examine the following differential equation :

$$u' = f(t,u_t), \tag{4.8}$$

where $f\in\mathbb{P}(\mathbb{R}\times C,\mathbb{R}^n)$. Will set $H(f)=\overline{\{\sigma(f,\tau)=f_\tau|\tau\in\mathbb{R}\}}$, where by stroke is noted closure in $\mathbb{P}(\mathbb{R}\times C,\mathbb{R}^n)$. Along with equation (4.8) will examine family of equations

$$v' = g(t,v_t), \tag{4.9}$$

where $g\in H(f)$. We will suppose, that function f is regular, i.e. whatever are $g\in H(f)$ and $\nu\in C$ equation (4.9) admits at least one solution $\varphi(\nu,g;t)$ defined on $[-r,+\infty)$. Will set $Y=H(f)$ and by $(Y,\mathbb{R},\sigma)$ will note a dynamical system of shifts on Y. Let $E=C\times Y$, and $\pi : E\times\mathbb{R}_+\to E$ is dynamical system on E defined by following rule : $\pi((\nu,g),\tau)=(\varphi_\tau(\nu,g),g_\tau)$, then triple $<(E,\mathbb{R}_+,\pi),(Y,\mathbb{R},\sigma),h>$ $(h=pr_2 : E\to Y)$ is nonautonomous dynamical system, where $\varphi_\tau(\nu,g)(\theta)=\varphi(\nu,g;\tau+\theta)(\theta\in[-r,0])$.

From general properties of solutions of equations (4.8) (see [5]) follows the following

Theorem 4.10. *Let $H(f)$ is compact in $\mathbb{P}(\mathbb{R}\times C,\mathbb{R}^n)$ and function f is bounded on $\mathbb{R}\times B$ whatever is set $B\subset C$, then nonautonomous dynamical system generated by equation (4.8) is asymptotical compact.*

Will say that set $I\subset C$ is invariant relatively to nonautonomous equation (4.8), if for any $v\in I$ exists $g\in H(f)$ and solution $\varphi(v,g;t)$ of equation (4.9) defined on $\mathbb{R}$ such that $\varphi_\tau(v,g)\in I$ for all $t\in\mathbb{R}$.

Will note by I_g set of all points from I through which passes at least one solution of equation (4.9) defined on $\mathbb{R}$ and lying entirely in I. Then obviously $I=\bigcup\{I_g : g\in H(f)\}$.

From general results of § 1-3 and theorem 4.10 follows a series of affirmations analogical to theorems 4.1-4.3

Will formulate one of affirmations of his kind.

Theorem 4.11. *Under condition of theorem 4.10 following conditions are equivalent :*

a. exists $R>0$ such, that $\overline{\lim}_{t\to+\infty} \mid\varphi(v,g;t)\mid< R$ for all $g\in H(f)$ and solution $\varphi(v,g;t)$ of equation (4.9) ;

b. equation (4.8) admits compact global attractor,i.e. exists such non-empty compact set $I\subset C$ invariant relatively to (4.8), that

b.1. for any $\varepsilon>0$ exists $\delta(\varepsilon)>0$ such that $\rho(v,I_g)<\delta$ implies $\beta(\varphi_t(v,g),I_{g_t})<\varepsilon$ for all $t\geq 0$;

b.2. takes place the equality

$$\lim_{t\to+\infty}\sup_{|v|\leq,g\in H(f)}\beta(\varphi_t(v,g),I)=0$$

for every $R>0$.

Remark. *Approach to study of nonautonomous differential equations by the point of view of general dynamical systems was suggested for the first time in works of Deysach L.G., Sell G.R. [31], Miller R.K. [32], Sell G.R. [33-34], Millionschikov V.M. [35] and Scherbakov B.A. [36]. This ideas are applied already more then three decades in works of many mathematicians for study of different classes of nonautonomous equations.*

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