Belitskii–Lyubich Conjecture for $\mathbb{C}$-Analytical Dynamical Systems

David Cheban

State University of Moldova, Republic of Moldova

E-mail: davidcheban@yahoo.com

In this talk the problem of global asymptotic stability of solutions for analytical dynamical systems (both with continuous and discrete time) is studied. In particularly, we present some new results for $\mathbb{C}$-analytical version of Belitskii–Lyubich conjecture. Some applications of this result for periodic $\mathbb{C}$-analytical differential/difference equations and the equations with impulse are given.

Belitskii–Lyubich conjecture [1].*Let E be a Banach space, $\Omega \subset E$ an open subset and $f : \Omega \mapsto E$ be a compact and continuously differentiable in Ω. Suppose D is a nonempty bounded convex open subset of X such that $f(\overline{D}) \subset \overline{D} \subset \Omega$ and $\sup\limits_{x\in\overline{D}} r(f'(x)) < 1$ ($r(A)$ is the spectral radius of linear bounded operator A). Then the discrete dynamical system $(\overline{D}, f)$, generated by positive powers of $f : \overline{D} \mapsto \overline{D}$, admits a unique globally asymptotically stable fixed point.*

Denote by $Hol(U, E)$ the set of all holomorphic functions $f : U \mapsto E$ equipped with the compact-open topology and by $Fix(f, D) := \{x \in D : f(x) = x\}$, where $D \subseteq E$ and $W^s(p) := \{x \in D : \lim\limits_{n\to\infty} \rho(f^n(x), p) = 0\}$ for all $p \in Fix(f, D)$.

Theorem *Let E be a complex Banach space, let U be a non-empty bounded domain in a Banach space E, $f \in Hol(U, E)$ be an asymptotically compact operator. Suppose that the following conditions hold:*

1. *D is a non-empty bounded convex open subset of U such that $\overline{D} \subset U$;*
2. *$f(\overline{D}) \subseteq \overline{D}$;*
3. *$r(f'(x)) < 1$ for all $x \in Fix(f, \overline{D})$.*

Then

1. *f has a unique fixed point $x_0 \in \overline{D}$;*
2. *x_0 is globally asymptotically stable, i.e., $W^s(x_0) = D$.*

Remark *Note that Theorem remains true if we replace the condition "$r(f'(x)) < 1$ for all $x \in Fix(f, \overline{D})$" by the following: there exists a fixed point $p \in Fix(f, \overline{D})$ such that $r(f'(p)) < 1$.*

References:

1. G. R. Belitskii and Yu. I. Lyubich. Matrix norms and their applications. *Operator Theory: Advances and Applications, 36. Basel etc.:* Birkhauser Verlag. viii, 209 p., 1988.