The Structure of Global Attractors for Non-Autonomous Gradient-Like Dynamical Systems

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In this paper we give the complete description of the structure of compact global (forward) attractors for non-autonomous perturbations of autonomous gradient-like dynamical systems.

Let $\Sigma \subseteq X$ be a compact positively invariant set, $\varepsilon > 0$ and $t > 0$.

The collection $\{x = x_0, x_1, x_2, \dots, x_k = y; t_0, t_1, \dots, t_k\}$ of the points $x_i \in \Sigma$ and the numbers $t_i \in \mathbb{R}_+$ such that $t_i \geq t$ and $\rho(x_i t_i, x_{i+1}) < \varepsilon$ $(i = 0, 1, \dots, k-1)$ is called a (ε, t, π)–chain joining the points x and y.

We denote by $P(\Sigma)$ the set $\{(x, y) : x, y \in \Sigma, \forall\, \varepsilon > 0\ \forall\, t > 0\ \exists\, (\varepsilon, t, \pi)$-chain joining x and $y\}$.

The point $x \in \Sigma$ is called chain recurrent (in Σ) if $(x, x) \in P(\Sigma)$.

We denote by $\mathfrak{R}(\Sigma)$ the set of all chain recurrent (in Σ) points from Σ.

Theorem 1. *Under some conditions the following statements take place:*

a) *for each $i \in \{1, 2, \dots, k\}$ and $\lambda \in [-\delta_0, \delta_0]$ by equality $\gamma^i_\lambda(y) := (\nu^i_\lambda(y), y)$ (for all $y \in Y$) is defined a continuous invariant section of non-autonomous dynamical system $\langle (X, \mathbb{R}_+, \pi_\lambda), (Y, \mathbb{Y}, \sigma), h\rangle$ ($h := pr_2$) generated by equation $u' = Au + f_0(u) + F(\sigma(t, y), u, \lambda)$ $(y \in Y)$, where $(Y, \mathbb{R}, \sigma)$ is a dynamical system on the metric space Y.*

b) *the dynamical system $(X, \mathbb{R}_+, \pi_\lambda))$ is compact dissipative and its Levinson center $J^\lambda = \bigcup\limits_{y \in Y} I^\lambda_y \times \{y\}$, where $\{I^\lambda_y : y \in Y\}$ is a compact global attractor of the cocycle φ_λ.*

c) *for each $i \in \{1, 2, \dots, k\}$ the set $\mathfrak{R}^\lambda_i := \mathfrak{R}(J^\lambda) \bigcap B[M_i, r_0]$ possesses the following properties:*

- *c1)* *$\mathfrak{R}^\lambda_i = \mathfrak{R}(\gamma^i_\lambda(Y))$, where by $\mathfrak{R}(Y)$ (respectively, $\mathfrak{R}(\gamma^i_\lambda(Y))$) is denoted the set of all chain recurrent points of dynamical system $(Y, \mathbb{R}, \sigma))$ (respectively, $(\gamma^i_\lambda(Y), \mathbb{R}, \pi_\lambda)$);*
- *c2)* *$\mathfrak{R}^\lambda_i$ is nonempty, closed and invariant;*
- *c3)* *$\mathfrak{R}(J^\lambda) = \coprod_{i=1}^k \mathfrak{R}^\lambda_i$.*

d) *if the set $\mathfrak{R}(Y)$ is chain transitive, then in the series of subsets $\mathfrak{R}^\lambda_1, \mathfrak{R}^\lambda_2, \dots, \mathfrak{R}^\lambda_k$ does not exit any l-cycle ($l \in \mathbb{N}$).*

e) *$J^\lambda_y = \bigcup_{i=1}^k W^u(\gamma^i_\lambda(y))$ for all $y \in Y$, where $W^u(\gamma^i_\lambda(y)) := \{x \in X :$ such that $h(x) = h(\gamma^i_\lambda(y))$ and there exists $\varphi \in \Phi_x$ such that $\lim\limits_{t \to -\infty} \rho(\varphi(t), \gamma^i_\lambda(\sigma(t, y))) = 0\}$.*

f) *if $(Y, \mathbb{R}, \sigma)$ is chain recurrent (i.e,. $Y = \mathfrak{R}(Y)$), then for each $y \in Y$ and $x \in \mathfrak{B}$ there exists a unique number $i \in \{1, 2, \dots, k\}$ such that $\lim\limits_{t \to +\infty} |\varphi(t, x, y) - \nu^i_\lambda(\sigma(t, y))| = 0$.*